

# Equalization of Waveguide Delay Distortion

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**Abstract**—Microwave all-pass circuits, consisting of reactive networks used in conjunction with wide-band circulators or couplers, are described. An extremely useful all-pass circuit for minimizing phase distortion results when the reactive network is a linear taper extending beyond cutoff. The delay characteristics of this circuit are well suited to the correction of the dispersive characteristics of TE or TM mode waveguides. Design formulas have been derived for the parameters of the tapers and a set of design curves is presented. The use of composite tapered sections and the use of tapers in conjunction with other equalizing circuits are described. Experimental results have been obtained for the phase of the reflection factor of a linear taper. Close agreement was observed between the results predicted by theory and the experimental data. Typical examples demonstrate that the time-delay variation of a length of uniform waveguide can be substantially reduced by linearly tapered waveguide equalizers.

## INTRODUCTION

THE BASIC relationships between network parameters and their amplitude and phase functions have been presented in a number of standard reference works [1], [2]. The fundamental relationship between the response of a network in the frequency domain (steady state response to applied sinusoidal signals) and the response in the time domain (transient response and response to complex signals expressed as a function of time) has also been clearly defined [2]–[4]. The time domain and frequency domain responses have been shown to be simply related through the Fourier or Laplace integral transforms of the complex (amplitude and phase) immittance functions of a network [3]–[5]. If the complex transfer function of a network is expressed in terms of its real (amplitude) and imaginary (phase) parts as

$$\log Z_i(j\omega) = \log G(\omega) + j\theta(\omega) \quad (1)$$

the time delay of a steady state signal at frequency  $\omega$  is given by [6], [7]

$$t_d(\omega) = - \frac{d\theta(\omega)}{d\omega} \quad (2)$$

This is identical to the time delay (group delay) of a transmission system,

$$t_d = \frac{d\beta(\omega)}{d\omega} \quad (3)$$

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where  $\beta$  is defined as the transmission phase shift. The difference in sign is a result of the difference in definition between the phase shift of a transmission system and the phase of a transfer function. If  $\theta$  (or  $\beta$ ) is in radians and  $\omega = 2\pi f$  is in radians per second,  $t_d$  is given in seconds. It is clear from (2) that when  $\theta$  vs.  $f$  is linear over a given bandwidth,  $t_d$  will be constant over that bandwidth. If the spectrum of a signal falls within that bandwidth, all its frequency components will be delayed uniformly by  $t_d$ . Since the same substitution  $t' = t + t_d$  can be made in all frequency components, the signal leaving the output port will be identical to that entering the input port except for a delay  $t_d$ , if the magnitude of the transmission coefficient is constant over that band. If  $t_d$  varies over the spectrum, the relative phasing of the various frequency components of the output pulse will differ from that of the input pulse so that distortion will occur. In that case, the time delay of the output signal cannot be strictly defined, but will effectively be a mean value of the time-delay function over the bandwidth containing the principal portion of the spectral energy.

In many microwave systems, the amplitude response is designed to be fairly constant over the band of frequencies corresponding to the principal spectrum of the applied signal, and is not usually as important a cause of distortion as the phase response. Therefore, the major interest is in phase correcting networks.

A matched two-port network having a constant amplitude response and a phase response function  $\beta_N(j\omega)$  can be equalized by a matched two-port network in cascade with the original network. The equalizing network must have a constant amplitude response and a phase response function  $\beta_E(j\omega)$  such that

$$\beta_N(j\omega) + \beta_E(j\omega) = K \cdot \omega \quad (4)$$

where  $K$  is constant. The time delay of the complete system, including the equalizing network, will then be

$$t_d = \frac{d\beta_T}{d\omega} = \frac{d(\beta_N + \beta_E)}{d\omega} = K. \quad (5)$$

A matched two-port network having a constant (unity) amplitude function and an arbitrary phase function  $\beta_E$  is called an "all-pass" network. In the microwave region, most signals can be accurately represented by a narrow spectrum about a fixed frequency carrier. Thus, the equalizing network need only approximate an all-pass network over this frequency spectrum.

The terms *phase equalizer* and *time-delay equalizer*

may be used interchangeably. In some problems it is simpler to think in terms of phase and in others of time-delay. However, the time-delay parameter is usually to be preferred, since it relates more directly to distortion.

All waveguides, except those uniform, lossless guides which propagate in the TEM mode, are dispersive, i.e., have nonlinear phase-vs.-frequency properties. The distortion of signals in dispersive lines has recently been the subject of much investigation. Beck [8] has described in a qualitative way the delay distortion effects of pulses in circular waveguide. Elliott [9] has considered the distortion of pulses in waveguide, and a more exact treatment by G. I. Cohn [10] was compared to Elliott's approximate analysis in a recent letter [11]. Similar work has been reported here and overseas [12]–[15].

The phase response of a uniform transmission line propagating in a TE- or TM-mode is given by

$$\beta = \frac{2\pi l}{\lambda_g} = \frac{2\pi l}{\lambda} \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}; \quad \lambda = \frac{c}{f} \quad (6)$$

where

- $l$  = length of transmission line
- $\lambda_g$  = guide wavelength
- $\lambda_c$  = cutoff wavelength
- $\lambda$  = wavelength in free space
- $c$  = velocity of light in the waveguide medium
- $\omega = 2\pi f$  = angular frequency in radians/s.

The time delay is then dependent upon the frequency and is given by

$$t_d = \frac{d\beta}{d\omega} = \frac{l}{c} \cdot \frac{\lambda_g}{\lambda} \quad (7)$$

A class of microwave networks capable of equalizing this time-delay variation over a significant portion of the waveguide frequency band will now be described.

#### EQUALIZERS USING LINEAR TAPERS

All-pass microwave networks can be realized by terminating the conjugate pair of arms of a wide-band 3-dB coupler in identical reactive networks. For transmission in one direction, an ideal circulator can be used with one arm terminated in the reactive network [16]–[19]. (See Fig. 1.) Two circulators and two identical reactive networks would be required for a reciprocal equalizer. When the reactive network is a length of waveguide, tapered so that the cutoff wavelength decreases with distance from the input port, characteristics well suited for the equalization of the dispersive characteristics of waveguide sections result. At any frequency, the tapered waveguide section can be considered to be propagating up to the point along the waveguide axis at which the waveguide cuts off. Thus, the time interval between the application of an incident

wave at the input port of the waveguide and the arrival of the reflected wave at that terminal increases with increasing frequency. The phase properties of either circuit are given by the negative of the angle of the reflection coefficient of the taper. The reflection factor of a tapered waveguide is described by the nonlinear Riccati differential equation. In general, this equation cannot be solved exactly. Existing approximate solutions to this equation are valid only when the reflection factor is small and slowly varying. In the present case, the structure is totally reflecting; therefore, existing approximate solutions cannot be employed.

As a first approximation to the phase of the reflection factor of a tapered line, it was assumed that 1) the taper is sufficiently gradual so that there are no reflections along the propagating section of the taper, 2) total reflection occurs at the point along the taper at which the cross section is exactly the cutoff dimensions of the guide, and 3) the total reflection at the cutoff point of the tapered waveguide is that caused by an open circuit at that point along the taper.<sup>1</sup>

The input reflection factor at the terminal plane  $T$  of Fig. 2 is equal to unity and has a phase angle  $-\phi$ , ( $\rho = e^{-j\phi}$ ). The phase shift introduced by such a section is  $\phi = 2\theta$ , where  $\theta$  is the electrical length of the propagating section of the taper. On the basis of the foregoing assumptions, the electrical length of a differential segment of the taper is given by

$$d\theta = \frac{2\pi}{\lambda_g(x)} dx \quad (8)$$

where the guide wavelength  $\lambda_g(x)$  is defined as the guide wavelength of a uniform waveguide whose cross section is identical to the cross section of the taper at  $x$ . Then the electrical length of a taper in the width of a rectangular guide is

$$\theta = 2\pi \int_0^l \frac{dx}{\lambda_g(x)} = 2\pi \int_0^l \frac{\sqrt{a^2 - \left(\frac{\lambda}{2}\right)^2}}{a\lambda} dx \quad (9)$$

where  $\lambda$  is the free space wavelength and  $a = a(x)$  is the width of the waveguide at a distance  $x$  from the input terminal  $T$ . Assume a linear taper such that  $a = a_0 - kx$ , where  $a_0$  is the width of the input guide and  $k$  is constant. Then integrating from the taper input terminal to the cutoff point ( $a = \lambda/2$ ) yields the phase shift

$$\phi = 2\theta = \frac{4\pi}{k\lambda} \left[ \sqrt{a_0^2 - \left(\frac{\lambda}{2}\right)^2} - \frac{\lambda}{2} \cos^{-1} \frac{\lambda}{2a_0} \right]. \quad (10)$$

<sup>1</sup> A similar approach was taken independently by Tang in deriving the shape of a taper necessary to produce linear delay [17]. The form of the solution reported here differs from that given by Tang. This new form leads to the wide-band equalization of waveguide runs by linear tapers.

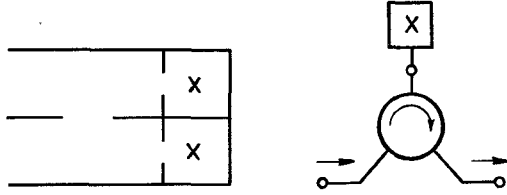


Fig. 1. Microwave all-pass networks.

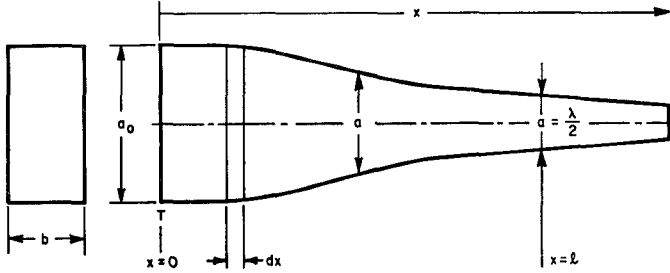


Fig. 2. Tapered rectangular guide.

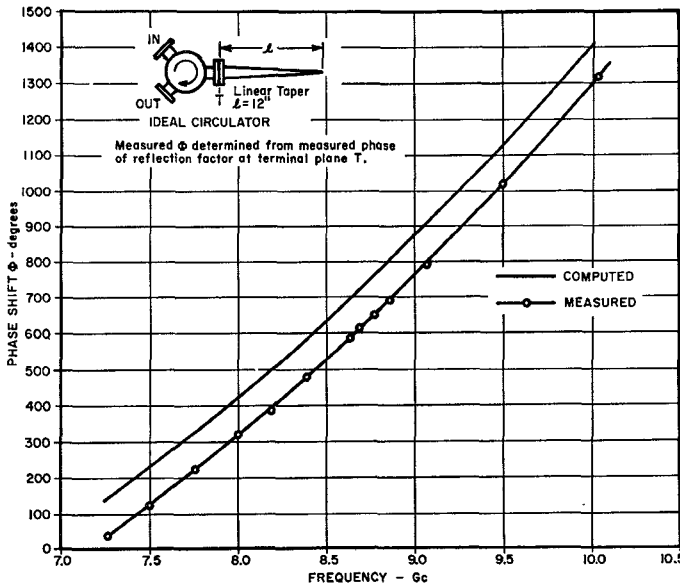


Fig. 3. Phase shift—linear tapered equalizer.

Measured values of the reflection factor phase for a particular taper are compared to the phase predicted by (10) in Fig. 3. The almost constant phase difference between the two curves indicates that the termination of the taper at the point of cut off is inductive, rather than the open circuit assumed in the derivation. This is in agreement with the characteristics of the  $TE_{10}$  mode impedance below cut off. The agreement in the slope of the phase characteristic demonstrates that (10) can be used in the design of phase and time-delay equalizers with good accuracy.

The time delay  $d\phi/d\omega$  for this network was found to have the simple form

$$t_d = \frac{2a_0}{kc} \frac{\lambda}{\lambda_{g0}} \quad (11)$$

where  $\lambda_{g0}$  is the guide wavelength in the input waveguide and  $c$  is the velocity of propagation in free space.

A comparison of the delay-vs.-frequency characteristics of a length of uniform waveguide and a length of tapered waveguide used in an equalizing network clearly indicates that the variation of one tends to compensate for the other. The total time delay of a length of waveguide  $L$  and an equalizer using a linearly tapered waveguide with taper slope  $k$  is given by

$$t_{dt} = t_{dw} + t_{dE} = \frac{L}{c} \frac{\lambda_{g0}}{\lambda} + \frac{2a_0}{kc} \frac{\lambda}{\lambda_{g0}} \quad (12)$$

where

$t_{dt}$  = total time delay

$t_{dw}$  = time delay of uniform waveguide of length  $L$

$t_{dE}$  = time delay of tapered equalizer.

The optimum correction will be obtained when  $t_{dt}$  exhibits a minimum in the operating band. The frequency at which minimum total time delay occurs is

$$f_m = f_c \sqrt{\frac{K}{K-1}} \quad (13)$$

where

$$K = 2a_0/kL; K > 1$$

$f_m$  = frequency of minimum total time delay

$f_c$  = cutoff frequency of input waveguide.

Conversely, if the frequency is specified at which the time delay is to be a minimum,

$$K = \frac{1}{1 - \left(\frac{f_c}{f_m}\right)^2} \quad \text{and} \quad k = \frac{2a_0}{L} \left[ 1 - \left(\frac{f_c}{f_m}\right)^2 \right]. \quad (14)$$

The value of the time delay at its minimum is found from (12) and (13)

$$t_{dtmin} = \frac{2L}{c} \sqrt{K} \quad (15)$$

and the relative change in time delay with respect to this minimum value is given by

$$\begin{aligned} \delta &= \frac{t_{dt} - t_{dtmin}}{t_{dtmin}} \\ &= \frac{1}{2\sqrt{K}} \left[ \frac{(1+K) - K\left(\frac{f_c}{f}\right)^2}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \right] - 1. \end{aligned} \quad (16)$$

Equation (16) can be rewritten to express the two band edge frequencies at which the time delay variation  $\delta$  is equal:

$$\begin{aligned} &\left(\frac{f_{1,2}}{f_c}\right)^2 \\ &= \frac{K}{(K-1) - 2\delta(2+\delta) \pm 2(1+\delta)\sqrt{\delta(2+\delta)}}. \end{aligned} \quad (17)$$

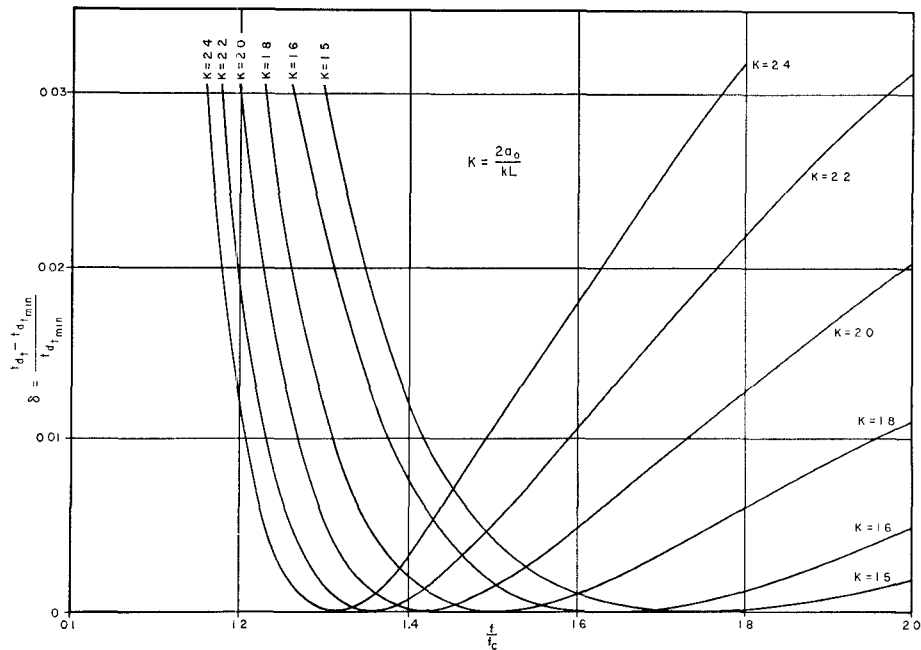


Fig. 4. Design data—tapered equalizer.

Equation (16) is plotted in Fig. 4 for different values of the parameter  $K$ .

#### TWO-SECTION TAPERED EQUALIZERS

Since the time delay of a linear tapered equalizer varies as  $\lambda/\lambda_g$ , it is apparent that the greatest contribution to time-delay correction occurs in the portion of the taper closest to cutoff at any operating frequency. One technique for reducing the overall size of a taper, therefore, would be to employ a two-section taper. The input section is a short length of line having a steep slope and ending in a width that is close to cutoff at the lower edge of the frequency band over which equalization is desired. A more gradual taper can then be employed beyond that point to introduce the maximum delay correction. (See Fig. 5.) The properties of a line containing such a double taper were analyzed.

From (9), with the appropriate change in variable, the phase shift introduced by a single tapered equalizer is

$$\phi = \frac{4\pi}{\lambda} \int_{\lambda/2}^{a_0} \frac{\sqrt{a^2 - \left(\frac{\lambda}{2}\right)^2}}{ka\lambda} da. \quad (18)$$

It was initially assumed that the taper was sufficiently gradual so that no reflection occurred in the propagating region. This same assumption was made for the line containing two tapered sections of different slopes. It was further assumed that reflections at the junction between the two tapers can also be neglected. In this case there are no singularities of the integrand over the range of integration; the integral can separate into two parts.

$$\phi = \frac{4\pi}{\lambda} \int_{\lambda/2}^{a_1} \frac{\sqrt{a^2 - \left(\frac{\lambda}{2}\right)^2}}{k_2 a \lambda} da$$

$$+ \frac{4\pi}{\lambda} \int_{a_1}^{a_0} \frac{\sqrt{a^2 - \left(\frac{\lambda}{2}\right)^2}}{k_1 a \lambda} da. \quad (19)$$

Similarly, the time delay can be determined from

$$\begin{aligned} t_d &= \frac{d\phi}{d\omega} = \frac{4\pi}{\lambda} \int_{\lambda/2}^{a_0} \frac{d}{d\omega} \left[ \frac{\sqrt{a^2 - \left(\frac{\lambda}{2}\right)^2}}{ka\lambda} \right] da \\ &= \frac{2}{c} \left[ \frac{1}{k_2} \int_{\lambda/2}^{a_1} \frac{ada}{\sqrt{a^2 - \left(\frac{\lambda}{2}\right)^2}} \right. \\ &\quad \left. + \frac{1}{k_1} \int_{a_1}^{a_0} \frac{ada}{\sqrt{a^2 - \left(\frac{\lambda}{2}\right)^2}} \right]. \end{aligned} \quad (20)$$

Taking the derivative under the integral sign is permissible, since the integrand of (19) is bounded and the derivative of the integrand is continuous over the range of integration. While one limit of integration is a function of  $\omega$ , it was also shown that there is no contribution to  $t_d$  resulting from this limit. Then, integrating (20) yields

$$t_d = \frac{2a_0}{ck_1} \frac{\lambda}{\lambda_{g_0}} + \frac{2a_1}{ck'} \frac{\lambda}{\lambda_{g_1}} \quad (21)$$

where

$c$  = velocity of light in free space

$\lambda$  = free space wavelength

$\lambda_{g_0}$  = guide wavelength in a uniform waveguide of width  $a_0$

$\lambda_{g_1}$  = guide wavelength in a uniform waveguide of width  $a_1$

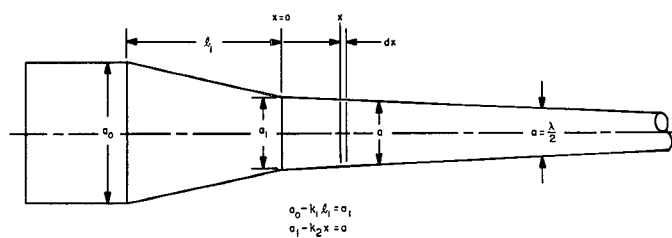
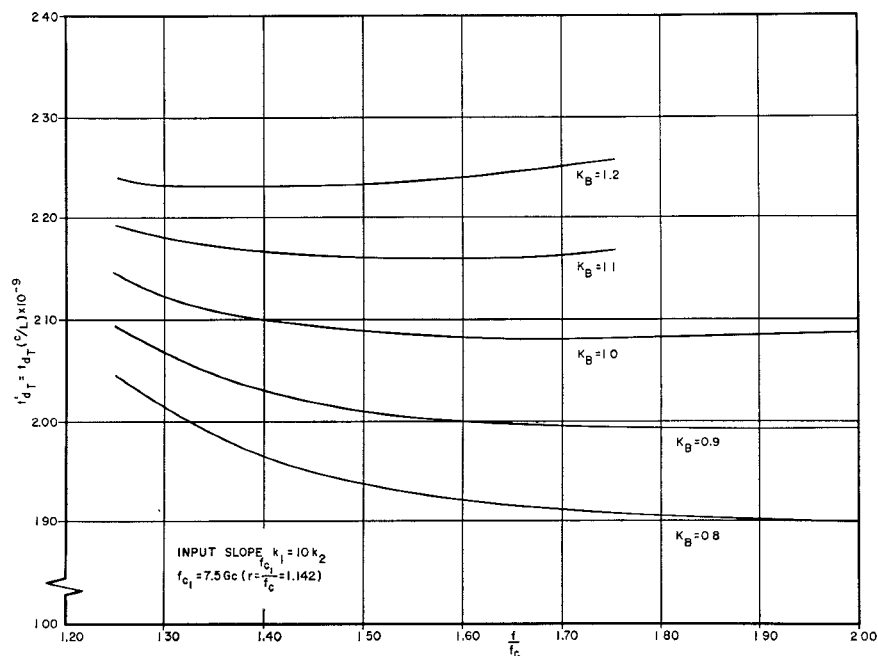
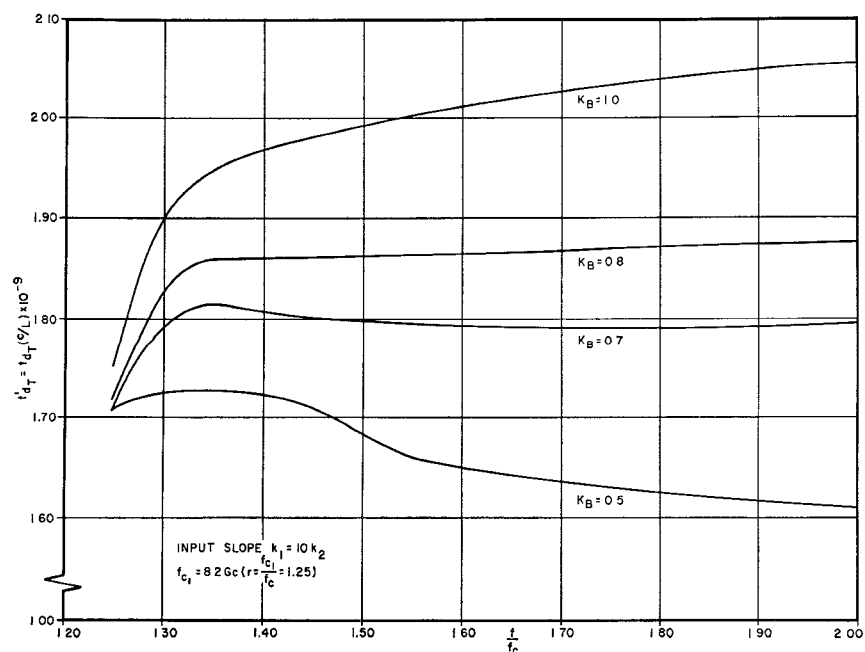


Fig. 5. Dual taper.

Fig. 6. Time-delay characteristics of equalizer with two-section taper —  $f_{c1} = 7.5$  Gc.Fig. 7. Time-delay characteristics of equalizer with two-section taper —  $f_{c1} = 8.2$  Gc.

and

$$\frac{1}{k'} = \frac{1}{k_2} - \frac{1}{k_1} \quad (22)$$

When the two-section tapered line is used to equalize the time-delay characteristics of a waveguide of length  $L$ , the total time delay is given by

$$t_{dt} = \frac{2a_0}{ck_1} \frac{\lambda}{\lambda_{g0}} + \frac{2a_1}{ck'} \frac{\lambda}{\lambda_{g1}} + \frac{L}{c} \frac{\lambda_{g0}}{\lambda} \quad (23)$$

Equation (23) can be rewritten in the form

$$t_{dt} = \frac{L}{c} \left[ \frac{1 + K_A - K_A \left( \frac{f_c}{f} \right)^2 + K_B \sqrt{1 - (1 + r^2) \left( \frac{f_c}{f} \right)^2 + r^2 \left( \frac{f_c}{f} \right)^4}}{\sqrt{1 - \left( \frac{f_c}{f} \right)^2}} \right] \quad (24)$$

where

$$\begin{aligned} r &= f_{c1}/f_c \\ f_c &= \text{cutoff frequency of waveguide of width } a_0 \\ f_{c1} &= \text{cutoff frequency of waveguide of width } a_1 \\ f &= \text{frequency} \\ K_A &= 2a_0/k_1L \\ K_B &= 2a_1/k'L \end{aligned}$$

The characteristics of the double-taper equalizer can best be appreciated by consideration of Figs. 6 and 7, where (24) is plotted for several different values of  $K_B$ . Both curves were based upon an assumed  $k_1$  ten times as great as  $k_2$ . In Fig. 6, the value of  $r = f_{c1}/f_c$  was taken to be 1.142. This corresponds to  $f_{c1} = 7500$  Mc/s in  $X$ -band waveguide. A value of  $f_{c1} = 8200$  Mc, resulting in  $r = 1.25$ , was assumed in the case illustrated in Fig. 7.

#### TYPICAL EXAMPLES

The degree to which the linear tapered waveguide equalizer can correct for the distortion introduced by a length of waveguide can best be illustrated by an example. A 100-foot length of WR-90 waveguide is assumed to operate over a frequency band from 8600 Mc/s to 9400 Mc/s ( $f/f_s = 1.31$  to 1.433). The delay introduced by a length of uniform guide is

$$t_{dw} = 1.01 \frac{\lambda_g}{\lambda} \text{ ns/ft.} \quad (25)$$

Using this relationship, the total time-delay variation of the 100-foot line over the given frequency range was found to be 14.98 ns. From Fig. 4 it can be seen that optimum flatness can be achieved over this band with an equalizer taper having a value of  $K \approx 2.2$ . The maximum time-delay deviation for  $K = 2.2$  will be  $\delta = 0.0015$ .

The minimum time delay of the combined line and equalizer circuit can be determined from

$$\begin{aligned} t_{d_{\min}} &= \frac{2L}{c} \sqrt{K} = \frac{2 \times 100(\text{ft}) \times \sqrt{2.2}}{3 \times 10^8(\text{m/s}) \times 3.281(\text{ft/m})} \\ &= 301 \text{ ns.} \end{aligned}$$

Thus, the maximum deviation of the equalized line will be 0.45 ns. The slope of the equalizer,  $k$ , is  $0.682 \times 10^{-3}$ . This results in a taper that goes from an initial width of 0.9 inch to 0 width in 110 feet. Since the maximum operating frequency is 9.4 Gc/s, no part of the taper beyond the width corresponding to a 9.4 Gc cutoff frequency will be above cutoff at any point in the operating band. Therefore, the taper need only extend a short distance beyond this point. Thus, it can be shown that the

equalizing taper in this case need only be slightly longer than 33.2 feet.

For the case illustrated, a taper is required to equalize a line which is approximately one-third of the length of the original line. Since both the transmission line and the equalizer are all-pass networks, it is possible to use a single taper at one point in the line, or alternately, to utilize individual tapered equalizers distributed along the line. For example, equalizers having tapers approximately 3.3 feet long can be placed at ten-foot intervals along the waveguide. This approach offers an additional advantage in that the slope of each taper is ten times greater than that required for a single tapered equalizer. Therefore, the tolerances upon taper fabrication are eased. However, ten circulators or hybrids will also be required.

Now, consider an equalizer, having the characteristics shown in the  $K_B = 1.2$  curve of Fig. 6, to correct the time-delay distortion of the same 100-foot line over the 8600 to 9400 Mc/s range. It can be demonstrated that a total line plus equalizer time delay of approximately 225 ns results. The variation over the band will be less than 0.3 ns.

Since the line to be equalized is an  $X$ -band waveguide, the width of the input line  $a_0 = 0.900$  inch. The curves of Fig. 6 are for  $r = 1.142$  so that  $f_{c1}$  in this case is 7500 Mc/s. Thus the width  $a_1$  is 0.787 inch, corresponding to the width of a uniform waveguide that cuts off at 7500 Mc/s. Then

$$k' = \frac{2a_1}{K_B L} = \frac{2 \times 0.787}{1.2 \times (100 \times 12)} = 1.092 \times 10^{-3}$$

From (22) and the assumption that  $k_1 = 10k_2$  it was determined that

$$\begin{aligned} k_1 &= 0.983 \times 10^{-2} \\ k_2 &= 0.983 \times 10^{-3} \end{aligned}$$

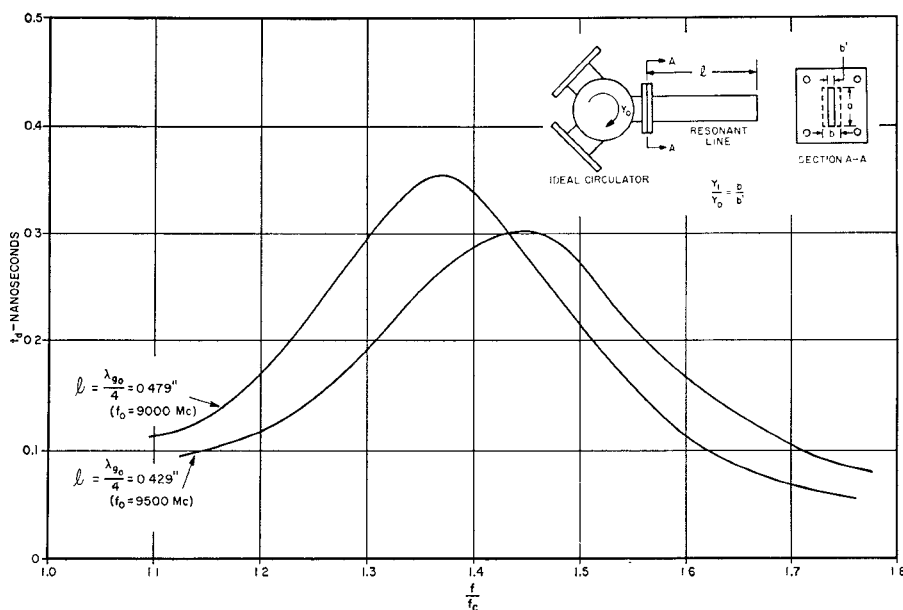


Fig. 8. Wide-band single-resonator equalizer.

The input taper length is then

$$l_1 = \frac{a_0 - a_1}{k_1} = 11.5 \text{ in.} = 0.97 \text{ ft.}$$

and the delay correcting taper length is

$$x = \frac{a_1 - \frac{\lambda_H}{2}}{k_2} = 161.7 \text{ in.} = 13.5 \text{ ft.}$$

where  $\lambda_H$  is the free space wavelength at the high-frequency edge of the band over which equalization is required. With an additional terminating length of approximately 2 feet, the total equalizer length is 16.5 feet. The advantages of a two-section taper are thus evident.

Still greater equalization over wide bands can be achieved by cascade connection of a supplementary all-pass circuit of the type shown in Fig. 8. The reactive network of the supplementary equalizer is a simple resonant cavity, and the time-delay characteristics of Fig. 8 are typical of single resonator equalizers. By comparing these curves to the equalized delay characteristics of Fig. 4, it can be seen that the supplementary equalizer will offer further reduction in time-delay variation.

### CONCLUSIONS

A considerable degree of equalization can be achieved by the use of simple linearly tapered waveguide sections in conjunction with a circulator or broadband hybrid junction. The length of the taper required for the degree of equalization considered above is approximately half the length of the line to be equalized. Since the region of the taper close to cutoff contributes the greatest part of the delay correction, tapers having differently shaped transitions into the cutoff region will afford a greater economy of taper length without seriously affecting the degree of delay correction obtained. Shorter tapers will also result when equalization is required over narrower portions of the waveguide band.

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